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***Inflation Targeting, Credibility and  
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# Inflation Targeting, Credibility and Confidence Crises\*

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## Abstract

We study the interplay between the central bank transparency, its credibility, and the inflation target level. Based on a model developed in the spirit of the global games literature, we argue that whenever *a weak* central bank adopts *a high degree of transparency and a low target level*, a bad and self confirmed type of equilibrium may arise. In this case, an over-the-target inflation becomes more likely. The central bank is considered weak when favorable state of nature is required for the target to be achieved. On the other hand, if a weak central bank opts for less ambitious goals, namely lower degree of transparency and higher target level, it may avoid confidence crises and ensure a unique equilibrium for the expected inflation. Moreover, even after ruling out the possibility of confidence crises, less ambitious goals may be desirable in order to attain higher credibility and hence a better coordination of expectations. Conversely, a low target level and a high central bank transparency are desirable whenever the economy has strong fundamentals and the target can be fulfilled in many states of nature.

Keywords: Inflation Target, Central Bank Transparency, Self-Confirmed Inflation.

JEL Classification: E58

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# 1 Introduction

Should central bank transparency go too far? How low the inflation target should be? We discuss such questions based on a model developed in the spirit of the global games literature. We conclude that a low target level and a high central bank transparency are desirable whenever the economy has strong fundamentals and the target can be fulfilled in many states of nature. On the other hand, if the central bank lacks credibility<sup>1</sup>, a higher target level and a lower degree of transparency is a better option.

Conclusions are explicit in some numerical exercises, in which the central bank chooses the target level of inflation and its level of transparency. If the actual central bank's choice is not well grounded, and does not consider the limitations imposed by the fundamentals and the credibility level, we say that a too ambitious framework is being adopted. In this case, confidence crises<sup>2</sup> may arise. Therefore, to manage an adequate inflation targeting regime, the trade-off between the ambitious framework and a more defensible one should be considered. Moreover, even when the economy is not subject to confidence crises, less ambition may be required in order to attain a higher credibility and hence a better coordination of expectations.

We start with the common uncertainty model, in which the uncertainty faced by the central bank is the same as the one faced by the private sector. The common doubt is about the future central bank's incentive in achieving the target. Based on this first approach, we conclude that a higher target for inflation increases the credibility in the precommitment stage, making the optimal target higher than the one obtained when this increasing credibility effect is not considered, as in the Cukierman-Liviatan model [5].

Second, extending the common uncertainty model to encompass confidence crises, we

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<sup>1</sup>Credibility is the extent to which agents believe that the central bank will carry out its pre-announced plan (the inflation target).

<sup>2</sup>By confidence crises we mean a new equilibrium of the self-fulfilling type where over-the-target inflation emerges in some states of nature.

find three self-confirmed type of equilibrium for the expected inflation: the target level, the discretionary level, and an inflation rate that is lower than the discretionary one, but higher than the target. These equilibria are named as optimistic, pessimistic and not-extreme, respectively. Once in the multiplicity region, as the target level becomes higher, the not-extreme equilibrium converges to the pessimistic one, not encouraging higher targets. On the other hand, if the target becomes high enough, multiple equilibria can be avoided and the optimistic one is ensured. The previous increasing credibility effect does not necessarily hold. The optimal target depends on the likelihood of each equilibrium to be selected and on the central bank's willingness to avoid confidence crises.

Third, we consider the no common uncertainty model. Again, the uncertainty is related to the future central bank's incentive in achieving the target. Once disclosed, incentive strength is denoted by the term commitment-strength. Therefore, the higher the commitment-strength is, the stronger the incentive disclosed. No-common feature comes from the fact that the central bank and the public compute the same expected commitment-strength, but the public perceive a wider range for its realization. Our assumption is that the difference in ranges is decreasing in the degree of the central bank transparency. In addition, if the central bank was fully transparent, the range difference would be zero and the uncertainty would be common. Results indicate that not only higher targets but also less transparency may help central bank to rule out confidence crises.

Finally, we extend the no common uncertainty model by perturbing common-knowledge not only between public and private sector, but also between private agents. Then, uniqueness can be ensured, even when speculative attacks are considered, as in Morris-Shin[6]. The first result is also recovered, i.e. a higher target for inflation increases the credibility in the precommitment stage. By contrast, in the presence of a *precise* public signal, the equilibrium multiplicity may still exist for a small lack of common knowledge, as in Angeletos and Werning[1]. In such a case, as the target level becomes higher, the not-extreme equilibrium converges to the pessimistic one. On the other hand, if the target

becomes high enough, multiple (bad) equilibria is avoided. Once again, the increasing credibility effect does not necessarily hold. The optimal target depends on the likelihood of each equilibrium to be selected over the multiplicity region as well as on the central bank's willingness to avoid a confidence crisis. Results also indicate that more precise public information may open the door to bad equilibrium, contrary to the conventional wisdom that more central bank transparency is always good when an inflation targeting regime is considered.

The aim of this last model is to provide a complete framework to study alternative central bank policies when the inflation targeting regime is considered. By complete, we mean a framework that encompasses both the recent debate about the information structure of the economies and its implications for the equilibrium multiplicity, as well as some classical features already discussed and understood in the rules versus discretion literature.

## **2 Common Uncertainty Model with Endogenous Credibility**

Our basic model follows Cukierman-Liviatan [5] which follows Barro and Gordon [2]. The Cukierman-Liviatan model appraises the uncertainty about the commitment enforcement and the conclusion is that the optimal target should be decreasing in the central bank credibility. This is an expected result, but it is derived considering a naive uncertainty approach, based on two central bank types: the “strong” one which always adheres to the announced policy and the “weak” one which does it only as an *ex-post* expedient. Moreover, the transparency issue cannot be totally addressed since each private agent has the same information set.

Then, we will gradually propose some extensions. First, the fulfillment of the target

may depend not on the central bank type, but on the intensity of a shock observed after the target announcement. Second, the commitment strength may also depend on the credibility, since the credibility itself should affect the central bank's decision about respecting (or not) the commitment. Finally, asymmetric uncertainty and the public strategic behavior should be considered. Next, we describe the Cukierman-Liviatan original model and further extend it to address the previous comments.

## 2.1 Agents and Timing Actions

There are two types of agents: the central bank and the private agent. Actions are taken in three stages: the central bank announces the target for inflation  $\pi_a$ , expectations  $\pi_e$  are formed by the public and actual inflation is chosen  $\pi$ . There are two central bank (sub)types  $i \in \{1, 2\}$  with different abilities to precommit. The strong type ( $i = 1$ ) always fulfills its commitment while the weak one ( $i = 2$ ) does it only if it is *ex post* expedient.

## 2.2 Central Bank Type

The objective function for the central bank of type  $i$  is positively related to surprise inflation and negatively related to actual inflation, as follows:

$$\begin{aligned} v^i(\pi_e, \pi_a) &= \max_{\pi \geq 0} A[\pi - \pi_e] - \frac{\pi^2}{2} - c^i(\pi_a, \pi) \\ c^i(\pi_a, \pi) &\equiv \begin{cases} k^i & \text{if } \pi_a \neq \pi \\ 0 & \text{otherwise.} \end{cases} \\ i &\in \{1, 2\}, \quad k^1 = A^2, \quad k^2 = 0 \\ A &> 0, \quad \pi_a \geq 0, \quad \text{and } \pi_e \geq 0 \end{aligned}$$

We add the cost of not fulfilling the target function  $c^i(\pi_a, \pi)$  to the original Cukierman-Liviatan model [5] to formalize that the strong type always fulfills the pre-announced target while the weak type is not concerned about the previous announcement.

Note that the central bank best response for actual inflation  $\pi^*$  is either the target level  $\pi_a$  pre-announced or the discretionary inflation level  $A$ . Let  $w_a^i$  be the welfare gain of type  $i$ , derived from keeping actual inflation on the target. Then,

$$\begin{aligned} w_a^i &= v^i(\pi_e, \pi_a) - v^i(\pi_e, A) \\ w_a^i &= k^i + f(A, \pi_a) \\ f(A, \pi_a) &\equiv A(\pi_a - A) - \frac{\pi_a^2}{2} + \frac{A^2}{2} \end{aligned}$$

, and it is easy to check that  $(w_a^1 > 0)$  and  $(w_a^2 \leq 0)$  for any  $\pi_a \in [0, A]$ . Both types of ability are justified for any possible target level, since one of the goals of the inflation targeting regime is to coordinate expectations from the discretionary inflation level  $A$  to socially optimal level 0.

### 2.3 Private Agent Type

There is a continuum of private agents without a strategic behavior. Their role is to process information, to form beliefs concerning the central bank's type and to compute the expected inflation. It is assumed that the private expectation about the central bank type is formed based on the exogenous probability  $\alpha$  of the type being strong ( $i = 1$ ). This probability measures the central bank's credibility. The expected inflation is given by:

$$E[\pi | \alpha, \pi_a] = \pi_e = \alpha \pi_a + (1 - \alpha)A$$

### 2.4 Result with Exogenous Credibility

Based on this framework, Cukierman and Liviatan [5] answered the following question: “*what should be the optimal announcement  $\pi_a^*$  for each type  $i$ ?*”. For  $(\alpha = 1)$ , the target and the expected inflation are the same. Then, the central bank type 1 promises and



delivers zero inflation rate. If we consider  $\alpha \in (0, 1)$ , the central bank type 1 promises and delivers  $A(1 - \alpha)$  inflation rate. As  $\alpha$  tends to zero the announcement effect on the expectations vanishes and the central bank type 1, who always keeps its promises, tends to pre announce the discretionary rate of inflation. Although type 2 ends up inflating at the discretionary rate, it has an interest to keep itself indistinguishable at the announcement stage in order to stimulate lower expectations:  $(\pi_e < A)$ . It follows that  $\pi_a^{*i} = A(1 - \alpha)$  for both types  $i$ . Accordingly, full credibility ( $\alpha = 1$ ) is not required for inflation targeting to be implemented. In the absence of precommitment, the result leads to an inflationary bias  $A$  that can be reduced whenever central banks are able to precommit with some credibility ( $\alpha > 0$ ). This bias reduction improves welfare. To totally eliminate the inflationary bias and to achieve the socially optimal inflation rate (zero), the ability to commit must not be only present but must also be undoubtedly recognized by the public. Otherwise, a lower inflationary bias reappears.

Next, we gradually extend this framework to argue that there are some other factors affecting the choice of the inflation target level.

## 2.5 Endogenous Credibility

Considering the endogenous credibility, we now compute not only the effect of the credibility on the target, but also the effect of the target choice on the credibility. It is important to consider this effect since less ambitious (*higher*) targets are attained more often than closer to the zero target, whenever the monetary policy is subordinated to fiscal financing requirements that can make more inflation tolerable during crisis times.

Therefore, we make the uncertainty about the commitment-strength coming not from some private suspicion related to the central bank type. There may be shocks that affect the cost of being above the target implying that *the central bank also faces uncertainty about the future inflation*.

The three-stage-model considered here is the same as the previous one, but with only one type of central bank, characterized by the cost function  $c(\pi_a, \pi)$ , which is common knowledge. Instead of being a real number, the cost of not fulfilling the target  $k$  follows a uniform distribution<sup>3</sup> with support  $[\underline{K}, \overline{K}]$ . It is drawn after the public's expectations have been formed but before the choice of the actual inflation. A low realization of  $k$  can be viewed as a shock that decreases the value of keeping the commitment without using short run effects from inflation. If we set  $(\overline{K} = 0)$  or  $(\underline{K} = A^2)$  the equilibrium can be computed as follows: with  $(\alpha^* = 0)$  and discretionary inflation rate, or with  $(\alpha^* = 1)$  and zero inflation rate, respectively. To keep attention on the intermediate case, where  $\alpha^* \in (0, 1)$ , we assume that  $\underline{K} = 0$  and  $\overline{K} = B > 0$ . Whether the commitment is delivered or not depends on the values of  $k$  and  $\pi_a$ . The credibility  $\alpha$  is given by:

$$\begin{aligned}\alpha(\pi_a) &= \text{prob}(w_a(k, \pi_a) > 0) \\ \alpha(\pi_a) &= \max \left\{ 1 - \frac{1}{B} \left[ A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]; 0 \right\}\end{aligned}$$

When choosing the target, the central bank understands that the higher is its level, the more credible its policy tends to be. In particular, only the  $A$ -inflation commitment is fully credible.

As in the previous model, because of the possibility of the cost of not fulfilling the target being positive, the commitment is listened by the public. Thus, commitment drives expectations and adds value to the economy. But now we have a different answer to what should be the optimal target  $\pi_a^*$ . On the one hand, for a given  $\alpha$ , the closer to zero is the target announcement, the lower the expected inflation is, since  $\pi_a$  drives it. This fact increases welfare. But, on the other hand, the closer to zero is the target announcement, the closer to the zero (or equal) the credibility  $\alpha$  is.

The endogenous credibility economy can be defined by two positive parameters, namely

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<sup>3</sup>Further, we also propose a similar framework based on uncertainty described by the normal distribution.

$A$  and  $B$ . The following proposition characterizes the equilibrium for the target announcement:

**Proposition 1** *For any economy  $\xi(A, B)$ , the equilibrium target  $\pi_a^*$  exists, it is unique, and it is in the interior of the set  $[0, A]$ . If we use the endogenous credibility  $\alpha(\pi_a^*(A, B))$  obtained and the same  $A$ , the solution to the original Cukierman-Liviatan model is a new optimal target  $\pi_a^{**}(A, \alpha(B))$ , always lower than  $\pi_a^*(A, B)$ . Proof: Appendix.*

Note that, for any fixed  $\pi_a$ , as  $B$  goes to infinite,  $\alpha$  goes to one. As  $B$  goes to zero,  $\alpha$  goes to zero too. Because of the increasing credibility effect from higher targets,  $\alpha(\pi_a^*)$  is bounded below by some positive value.

A less ambitious target improves credibility in the announcement and induces positive welfare effect for economies where the central bank is not able to set “fully-credible” commitment. Then, when setting targets, the central bank must be aware not only that the announcement effect on expectations is reduced by credibility but also that the announcement itself affects credibility. We present in the Figure 1 the optimal target announced when considering the credibility effect versus the one announced when this effect is neglected. Note that, the weaker is the ability in precommitment (lesser  $B$ ), the higher the difference between the two announcement values is. On the other hand, as  $B$  increases, the credibility  $\alpha^*$  becomes closer to one and the difference between  $\pi_a^*$  and  $\pi_a^{**}$  becomes smaller.

### 2.5.1 Self-Confirmed Inflation

One possible reason to assume some cost for not fulfilling the target comes from the fact that the public may learn from the central bank’s decisions. In this sense, credibility loss is a punishment for abandoning the target. Then, credibility  $\alpha$  may affect the central bank’s incentive in defending the target. Moreover, the value of respecting the target should be increasing in the current credibility whenever the benefits from “keeping the

target decisions” are computed in a much slower way than the credibility loss associated with “not keeping the target decisions”. This feature opens the door for confidence crises and self-confirmed inflations.

Keeping the three-stage framework, the economy  $\xi^{sci}$  with self-confirmed inflation is now defined by:  $\{A; \pi_a; \epsilon; n; h(\cdot)\}$ , with  $A > 0$ ,  $\pi_a \in [0, A]$ ,  $\epsilon \geq 0$ ,  $n \in \mathbb{R}$ , and  $h$  defined as an increasing function that maps the credibility  $\alpha \in [0, 1]$  into a real number  $h(\alpha)$ . In order to reach a simple characterization of the equilibrium, we also assume that  $h$  is a linear function. All of them are common knowledge. The central bank type is unique and it is defined again by  $[\underline{K}, \overline{K}]$ , but now, to be more general, we set  $\underline{K} = (n - \epsilon)$  and  $\overline{K} = n$ . The objective function for the central bank is the same as the one considered in the first-proposition model, except for the cost of not keeping the target function, now defined as follows:

$$c^i(\pi_a, \pi, \alpha) \equiv \begin{cases} k+h(\alpha) & \text{if } \pi_a \neq \pi \\ 0, & \text{otherwise.} \end{cases}$$

where  $k$  is a random variable distributed according to  $U[n - \epsilon, n]$ ,  $\alpha^*$  is the endogenous credibility that solves  $\alpha = \text{prob}(\pi = \pi_a | \alpha)$ , and the function  $h$  measures how much the cost of not keeping the target depends on the public expectations. The timing of actions is also the same: a target  $\pi_a$  is announced, expectations  $\alpha^*$  are formed, the uncertainty  $k$  is solved and the actual inflation  $\pi$  is implemented.

With such assumptions, both fundamental and expectations shocks are important to compute the central bank’s incentives in choosing the actual inflation. The welfare gain  $w_a$  from keeping the target can be written as follows:

$$\begin{aligned} w_a(k, \alpha) &= k - x(\pi_a) + h(\alpha) \\ x(\pi_a) &\equiv A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2} = -f(A, \pi_a) \\ \text{with } \frac{dx(\pi_a)}{d\pi_a} &\leq 0 \end{aligned}$$

It is always possible to reach an equilibrium  $\alpha^*$  for any economy  $\xi^{sci}$  and it may be possible to reach more than one. When the uncertainty about the future central bank's incentives is high, i.e.  $\epsilon > h(1) - h(0)$ , we classify the economy  $\xi^{sci}$  as a  $\xi^u$ -type economy, otherwise, we classify the economy as  $\xi^m$ -type. The following proposition characterizes the equilibrium:

**Proposition 2** *The economy  $\xi^{sci}$  always admits an equilibrium. For a  $\xi^u$  type economy there are three types of unique- $\alpha^*$ -equilibrium, depending on the target level  $\pi_a$ :*

$$\begin{aligned} [\alpha^* = 1] &\equiv \text{perfect commitment} && \Leftrightarrow x(\pi_a) \leq (n - \epsilon + h(1)) \\ [\alpha^* \in (0, 1)] &\equiv \text{imperfect commitment} && \Leftrightarrow x(\pi_a) \in (n - \epsilon + h(1), n + h(0)) \\ [\alpha^* = 0] &\equiv \text{discretionary "commitment"} && \Leftrightarrow x(\pi_a) \geq n + h(0) \end{aligned}$$

; for  $(\xi^m)$  type economy it is possible that:

$$\begin{aligned} x(\pi_a) &\leq (n - \epsilon + h(1)) \text{ and perfect commitment is possible} \\ x(\pi_a) &\in (n + h(0), n - \epsilon + h(1)) \text{ and any commitment-type is possible} \\ x(\pi_a) &\geq n + h(0) \text{ and discretionary "commitment" is possible.} \end{aligned}$$

*Proof: Appendix.*

According to this proposition, if there is too much uncertainty concerning future central bank's incentives,  $\epsilon > h(1) - h(0)$ , then the equilibrium is unique: perfect commitment for very strong central bank (high  $n$ ), discretionary for very weak central bank and imperfect commitment otherwise. When  $\epsilon$  is sufficiently large, there is no room for self-confirmed inflation.

On the other hand, if the region for the possible central bank's incentives shrinks ( $\epsilon$  decreases), the uniqueness remains only for a very strong or for a very weak central bank. The intuition is that some economies may be subject to multiple equilibria when the

decision about respecting the target or not depends much on the credibility ( $\alpha$ ) before the realization of  $k$ . In such a case, increasing the target for inflation may have two welfare effects. First, and the new one, it is possible that only perfect commitment equilibrium remains. Second, as long as  $h'(\cdot) > \epsilon$ , the *critical*  $k^*$ <sup>4</sup> becomes greater when the target is increased, and hence the state region for good expectations ( $\pi = \pi_a$ ) shrinks. Then, the central bank credibility may be increasing in the target or not, whenever  $\epsilon < h(1) - h(0)$ .

As we have shown in the Figure 2, when  $h(\alpha) \equiv \rho\alpha$  and multiple equilibria are possible, increasing the target *may* be a good deal if it avoids multiplicity. But this decision also depends on the central bank's willingness to avoid a confidence crisis and on the probability of each equilibrium to be selected over the multiple equilibria region. Coordination failure allows all possible equilibria to occur. Such difficulty is usually solved by the definition of an arbitrary sunspot variable in order to compute expected welfare for each policy choice<sup>5</sup>. Obviously, the policy recommendations varies accordingly to the assumptions about the sunspot distribution, so we avoid an equilibrium selection theory. Instead, the expected welfare for each equilibrium versus policy variables are plotted on the same figure. In this way, the reader can conclude by himself the best option for some policymaker, as a 'max-min' type, for example.

Besides increasing the target level, an alternative public policy to rule out confidence crises may be related to the availability of the public information. In the numerical exercise presented in the Figure 3, we consider that the central bank can add some noise  $\eta$  to the private information, by being less transparent. In this case, the  $k$ -distribution perceived by the central bank is uniformly distributed on  $[n - \epsilon, n]$ , but the  $k$ -distribution perceived by the public is uniformly distributed on  $[n - \epsilon - \eta, n + \eta]$ , with ( $\eta \geq 0$ ). Results show that less transparency may avoid confidence crises when the target level considered is equal to 3%.

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<sup>4</sup> $k^*$  solves:  $k = x - h\left(\frac{n-k}{\epsilon}\right)$ .

<sup>5</sup>See Cole and Kehoe [4] for coordination failure in the public debt market.

### 3 No-Common Uncertainty Model

In this section, to appraise the information issue in a more sophisticated way, we consider the public as a set formed by different private agents. Up to this point, each private agent's role has been to process the same information, to form the same beliefs concerning the central bank's incentives and to compute an unique inflationary expectations. By adding strategy options (to attack or not the target) and a payoff structure to them, and assuming an asymmetric information structure, a coordination motive may arise from some strategic complementarity in their actions. Moreover, since the attack-mass will depend on the endogenous credibility defined by  $prob(\pi = \pi_a | \text{public information})$ , this model also provides one possible interpretation for the function  $h(\cdot)$ .

The new economy  $\xi^{ai}$  with an asymmetric information structure is defined as a one-shot game with two stages, and two agent types characterized by:  $\{A, \pi_a, \rho, h(\cdot), k, c, \sigma, \sigma_p\}$  with  $A > 0, \pi_a \in [0, A], \rho > 0, h(\alpha) \equiv \rho\alpha, k \in \mathbb{R}, c \in (0, 1), \sigma > 0, \sigma_p > 0$ .  $\sigma$  and  $\sigma_p$  define the information structure and  $c$  defines the payoff structure, both of them related to each private agent.  $\{A, \pi_a, \rho, h(\cdot), k\}$  define the central bank type. In the last stage of the game the central bank chooses the actual inflation  $\pi$ , after observing the speculative actions  $(1 - \alpha)$ , which is taken in the first stage.

#### 3.1 Private Agent Type

The population of private agents (speculators) is continuous and normalized to unit. Each speculator  $j$  may set  $\alpha^j$  equal to one or zero. If she sets  $\alpha^j$  equal to zero she believes that the target will probably<sup>6</sup> not be reached. With some cost, she speculates based on her beliefs (buying foreign currency, for example). If she sets  $\alpha^j$  equal to one she believes that the target will probably be reached. In this case, she does not bet against the central bank (keeping savings denominated in local currency, for example). Then, the size of the

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<sup>6</sup>In equilibrium, with probability higher than  $(c)$ .

attack  $(1 - \alpha)$  is given by  $(1 - \text{prob}(\alpha^j = 1))$ . Each  $j$  payoff is defined as being equal to  $(1 - \alpha^j)(g_s - c)$ . The speculative gain  $g_s$  depends on the central bank's response. If the target is sustained, then  $g_s = g_a$ , otherwise  $g_s = g_A$ , where  $(g_A > c > g_a)$ . With this payoff structure, to speculate is a good deal only when the target is abandoned, since  $(g_A - c > 0)$ , and  $(g_a - c < 0)$ . To keep our framework as close as possible to the one proposed in Angeletos and Werning [1], we define  $g_A \equiv 1$ , and  $g_a \equiv 0$ , and we also consider that the strength of the status-quo  $k$  is not common knowledge. Instead of observing the realization of the  $k$ -value, each player  $j$  observes the public signal  $s^p$  and the private signal  $s^j$ ,

$$\begin{aligned} s^j &= k + \sigma \varepsilon_j ; \sigma > 0 \text{ and } \varepsilon_j \sim N(0, 1) \\ s^p &= k + \sigma_p \varepsilon_p ; \sigma_p > 0 \text{ and } \varepsilon_p \sim N(0, 1), \text{ where:} \end{aligned}$$

$\varepsilon_j$  is assumed to be independent of  $k$  and  $\varepsilon_{j'}$  for all  $j' \neq j$ .  $\varepsilon_p$  is also assumed to be independent of  $k$  and  $\varepsilon_j$ .

## 3.2 Central Bank Type

The objective function for the central bank is the same as the one considered in the second-proposition model. Then, central bank keeps the inflation equal to the target  $\pi_a$  if and only if  $(w_a \geq 0)$ <sup>7</sup>. Otherwise it inflates at level  $A$ .  $k$  is drawn in the beginning of the game from the support of the improper uniform, defined over the entire real line, but its value is not observed directly by the public (speculators) as previous explained.

Since the target tends to be abandoned during intense attacks, the incentive to attack is increasing in the size of the attack. Note also that, the greater is the size of the attack, the lower the endogenous credibility is.

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<sup>7</sup> $w_a \equiv k + \rho\alpha - x(\pi_a)$  and  $x(\pi_a)$  is given by  $A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}$ . The no-attack-mass defined by  $(\alpha)$  is increasing in the aggregate credibility, which is defined by  $\text{prob}(\pi = \pi_a | \text{public information})$ .



### 3.3 The Equilibrium

Results are based on monotone equilibria defined as perfect Bayesian. For each public signal, the agent  $j$  attacks if and only if her private signal  $s^j$  is less than some threshold  $s^*(s^p)$ . The mass of agents that ends up attacking is given by:

$$prob(s^j < s^*(s^p, \pi_a) | s^p, k, \pi_a) = \Phi\left(\frac{s^*(s^p, \pi_a) - k}{\sigma}\right) = 1 - \alpha$$

where  $\Phi(\cdot)$  denotes the cumulative distribution function for the standard normal. The central bank sustains the target if and only if  $k$  is greater than  $k^*$ , which is given by:

$$k^*(s^p, \pi_a) = x(\pi_a) + \rho \cdot \Phi\left(\frac{s^*(s^p, \pi_a) - k^*(s^p, \pi_a)}{\sigma}\right) - \rho$$

The expected payoff from attacking must be equal to zero whenever  $s^j = s^*(s^p, \pi_a)$ <sup>8</sup>, which implies the following indifference condition:

$$\sqrt{\tau} \cdot \Phi^{-1}(c) = k^*(s^p, \pi_a) - \frac{\tau s^*(s^p, \pi_a)}{\sigma^2} - \frac{\tau s^p}{\sigma_p^2}, \text{ where } \tau = \frac{\sigma_p^2 \sigma^2}{\sigma_p^2 + \sigma^2}$$

which, after replacing  $s^*(k^*)$ , becomes:

$$\Phi^{-1}\left(\frac{k^* + \rho - x(\pi_a)}{\rho}\right) = \frac{\sigma}{\sigma_p^2} [k^* - s^p] + \frac{\sigma}{\sqrt{\tau}} \Phi^{-1}(1 - c)$$

It is always possible to find at least one  $k^* \in [x - \rho, x]$  that solves the previous equation. This solution is unique for every public signal ( $s^p$ ) if and only if  $\sigma \in (0, \frac{\sigma_p^2 \sqrt{2\pi}}{\rho}]$ .

According to Angeletos-Werning [1], for any (positive) doubt related to the public signal  $\sigma_p$ , uniqueness is ensured by a sufficiently small (positive) doubt related to the private signal  $\sigma$ . Moreover, multiplicity may vanish when the common knowledge is perturbed, as in Morris and Shin [6]. This result always holds for some exogenous information

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<sup>8</sup>  $s^*(s^p, \pi_a) = \sigma \cdot \Phi^{-1}\left(\frac{k^*(s^p, \pi_a) + \rho - x}{\rho}\right) + k^*(s^p, \pi_a)$

structure because precise private information anchors individual behavior and makes it difficult to predict the actions of others. Under the reasonable assumption that the improvement in the private signal implies improvement in the public signal, it is possible that public information becomes more precise faster than the private one, and so multiplicity may still exist even for small common knowledge perturbation ( $\sigma \rightarrow 0$ ). In this case, the public signal drives individual behavior more than the private signal, motivating mass movements.

By keeping the exogenous information structure, it is possible to set multiple-equilibria economies  $\xi^m$  when  $\sigma > \frac{\sigma_p^2 \sqrt{2\pi}}{\rho}$  and unique-equilibrium economies  $\xi^u$  when  $\sigma \in (0, \frac{\sigma_p^2 \sqrt{2\pi}}{\rho}]$ .

**Proposition 3** *For  $\xi^u$  type economy, a higher target increases the commitment credibility. For  $\xi^m$  type economy, a higher target may turn the commitment more credible or not. The effect on the credibility will depend on the likelihood of each equilibrium  $k^*(s^p)$  to be selected. Proof: Appendix.*

When the target is increased, two effects are observed. First, the shock required for the commitment to be abandoned becomes greater (smaller  $k$ -realization), for any fixed  $\alpha \in (0, 1)$ . This fact inhibits attacks and adds credibility. Second, as the central bank sets a higher target for inflation, new attack-strategies (or beliefs) are settled and this fact may increase the attack mass (decrease the credibility) because  $\alpha^*(\pi_a)$  may be decreasing in  $\pi_a$ . In this case, a higher target gives more room for over-the-target inflation. The first effect is always preponderant for a  $\xi^u$  type economy.

For the  $\xi^m$  type economy, the first effect tends to be preponderant when the extreme equilibria are selected ( the  $k^*(s^p)$  closest to  $x$  (or to  $x - \rho$  ) ). Note that, when strategies are too optimistic or too pessimistic, the size of an attack is closer to zero or to one, respectively. For more pondered strategies, based on the not-extreme equilibrium  $k^*(s^p)$ , the attack-mass and the no-attack-mass are both significant. Then, enlargement in the attack size induced by more aggressive strategies is more relevant and the critical

$k$  becomes greater for higher targets (see Figure 4). When the not-extreme equilibrium tends to be selected, relaxing the target in order to attain more credibility is a good idea only if multiplicity is avoided in many states  $s^p$ . Otherwise, the speculative movement could be strengthened and the commitment enforcement, reduced. In the Figure 5 we show extreme  $k$ -equilibria as a function of the public signal realization  $s^p$ . The parameters used are the same as those used in the Figure 4 and the target level considered in the benchmark case is equal to 2%. Note that, either in the benchmark case with a higher target level (benchmark parameters except for  $\pi_a = 12\%$ ) or in the benchmark case with a lower transparency level (benchmark parameters except for  $\sigma_p = 35\%$ ), multiplicity vanishes in our numerical approximation. On the other hand, when the target is increased from 2% to 6.5%, multiplicity can be noted<sup>9</sup>.

### 3.4 Central Bank Transparency and Welfare Analysis

A lower  $\sigma_p$  value may be viewed as more central bank transparency. According to our results, more precise public information may open the door to bad equilibrium, contrary to the conventional wisdom that more transparency is always good in an inflation targeting framework. Some other papers have argued in the same direction, but based on different models. In Metz [3], more precise public information increases the likelihood of currency crises in case of bad fundamentals. Morris and Shin ([7] and [8]) have pointed out that welfare effect of increased public disclosures is ambiguous and that there is a dilemma between managing market prices and learning from market prices. They also conclude that when a Central Bank cannot actually control inflation<sup>10</sup>, the inflation targeting regime could fail and undermine credibility. In this sense, it would be better for the central bank to simply forecast inflation and point out the extent to which its forecasts are contingent on

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<sup>9</sup>For sufficiently high target, theoretical multiplicity becomes negligible. In this case, to increase the target up to 6.5% is not sufficient.

<sup>10</sup>Sargent and Wallace [9] is a good reference for the limitations of the central bank's control over inflation.

fiscal policy. Our results suggest that inflation targeting may be a good set-up whenever the central bank can actually control *some* level of inflation in *some* states of nature.

In the Figure 6 we present the combined welfare effect of a higher target and less transparency, considering that the central bank knows its commitment strength in the beginning of the game, which is given by ( $k_o = 3\%$ ). Note that, for any given  $k$  and  $\pi_a \in [0, A]$ , the expected “aggregated” welfare can be computed in the following way:

$$E \max_{\pi(s^p)} \left[ A (\pi(s^p) - \pi_e(s^p)) - \frac{\pi(s^p)^2}{2} - c(\pi; \pi_a, s^p) \right]$$

$$\pi^e \equiv \Phi(s^{p*}) \cdot A + [1 - \Phi(s^{p*})] \cdot \pi_a$$

$$c(\pi; \pi_a) = \begin{cases} k + \rho \cdot (\alpha^*(s^p)) & ; \pi \neq \pi_a \\ 0 & ; \text{otherwise.} \end{cases}$$

and the aggregated uncertainty is given by  $s^p$ . We plot in the vertical axis the expected welfare cost for inflation only for the extreme equilibria cases. We can observe that, as the central bank becomes less transparent, the welfare associated with the optimistic equilibrium is reduced, while the welfare associated with the pessimistic equilibrium is increased. Results also indicate that less transparency may be welfare improving out of the multiplicity region. For a lower target level, 2%, this welfare effect is present over a wider “unique-equilibrium-region”, but on the other hand, it is flatter than the equivalent effect observed under a higher inflation target level, 6.5%. Finally, for  $\sigma_p = 0.19$ , we verify that multiplicity is ruled out from the numerical approximation when the target is increased from 2% to 6.5%.

In the Figure 7, we replicate the result from the Figure 6, but considering a very strong central bank ( $k_o = 30$ ). In this case, uniqueness is ensured and we can say that a higher transparency increases the expected welfare, contrary to the result just presented for a weak central bank ( $k_o = 3\%$ ).

## 4 Concluding Remarks

We first appraise how the target level of inflation should be set in the presence of uncertainty about the ability in precommitting. Ruling out *confidence crises and imperfect information*, we conclude that higher target for inflation increases the credibility in the precommitment stage.

Second, adding the possibility of confidence crises under perfect information, we conclude that to set a higher target for inflation may stimulate over-the-target inflation and reduce the central bank credibility. On the other hand, multiple bad equilibria may be avoided. The optimal target will depend on the likelihood of each equilibrium to be selected and on the central bank's willingness to avoid confidence crises.

Third, we rule out confidence crises again, but now by breaking common knowledge with exogenous and imperfect information structure, as in Morris and Shin [6]. In this case, it is possible to conclude that a higher target for inflation increases the credibility in the precommitment stage.

Finally, in the presence of a precise public signal, confidence crises may still exist even for a small lack of common knowledge, as in Angeletos and Werning ([1]). In this case, precise public information may open the door to bad equilibrium. Once again, in the multiple equilibria case, to set higher targets for inflation may stimulate over-the-target inflation and reduce the central bank credibility.

Therefore, results can be resumed as follows: depending on the perception of the *target-strength* uncertainty, it may be optimal to have an *ideal* status-quo (low target under high transparency) or a *more defensible* one (higher target and less central bank transparency). The optimal policy will also depend on the central bank's willingness to avoid confidence crises.

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## 5 Figures

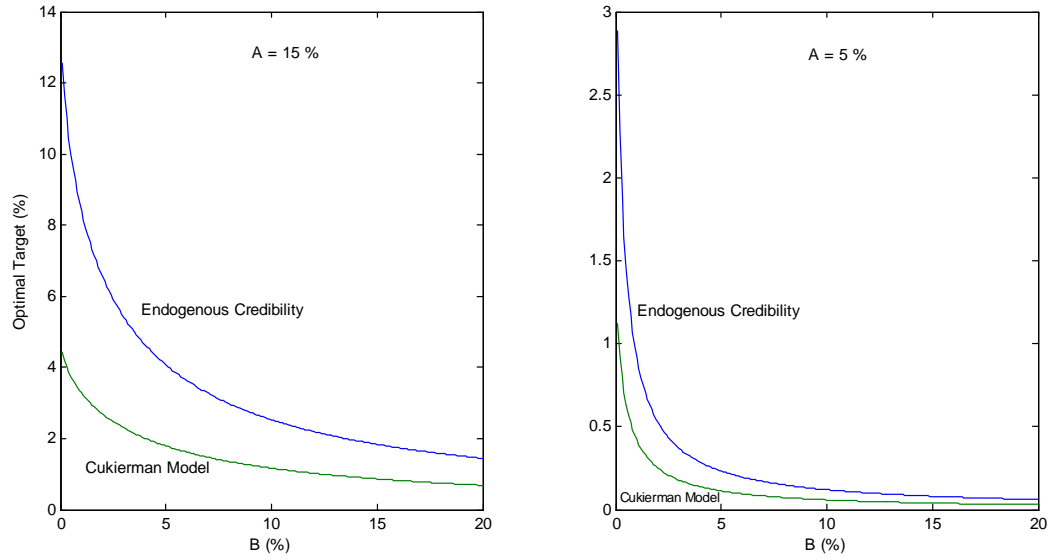


Figure 1: Optimal Targets

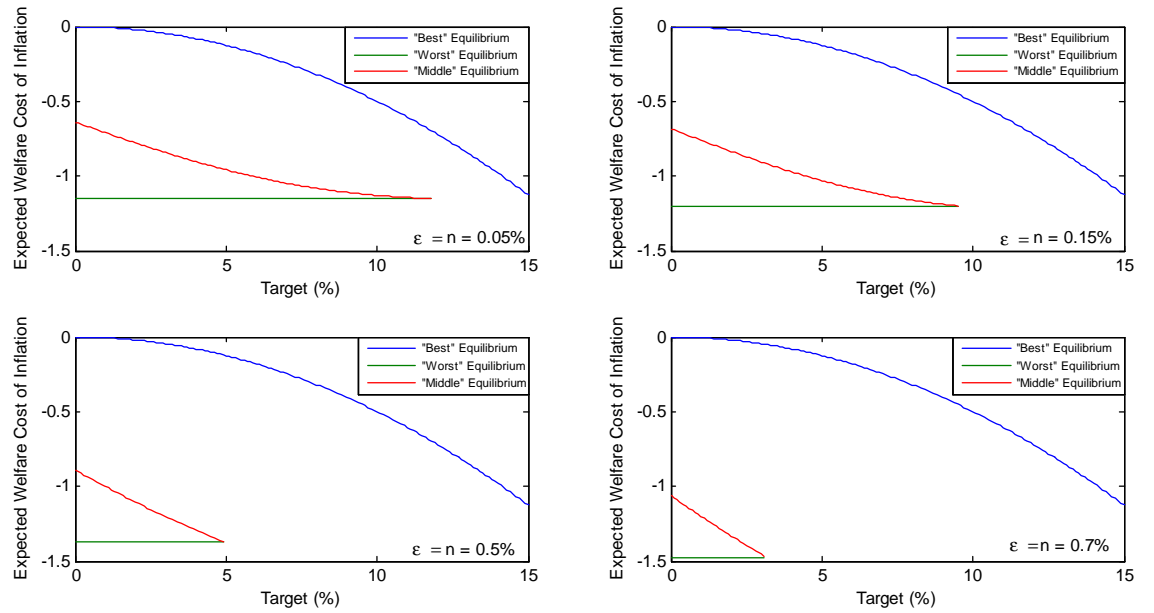


Figure 2: Self-Confirmed Equilibria ( $A = 15\%$  ;  $\rho = 2.5\%$ )

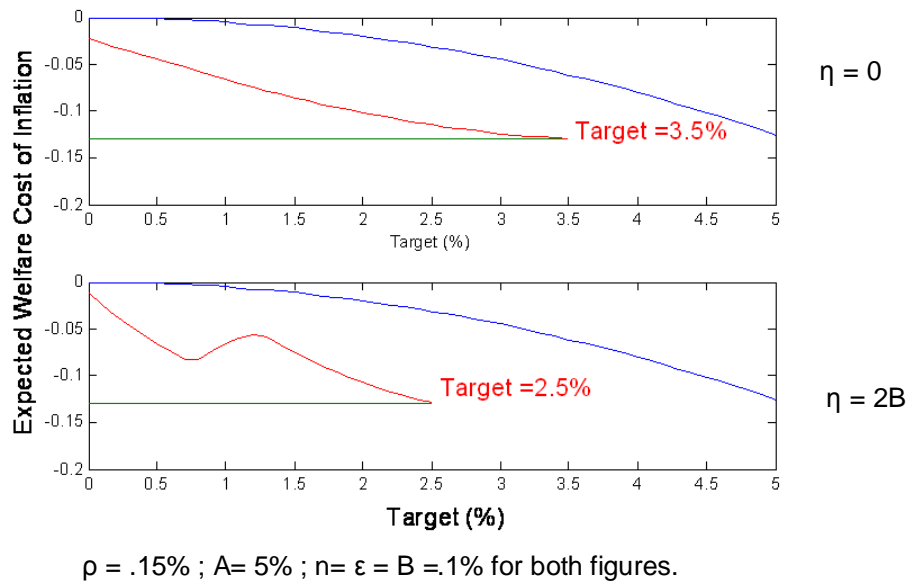


Figure 3: Multiplicity and Transparency



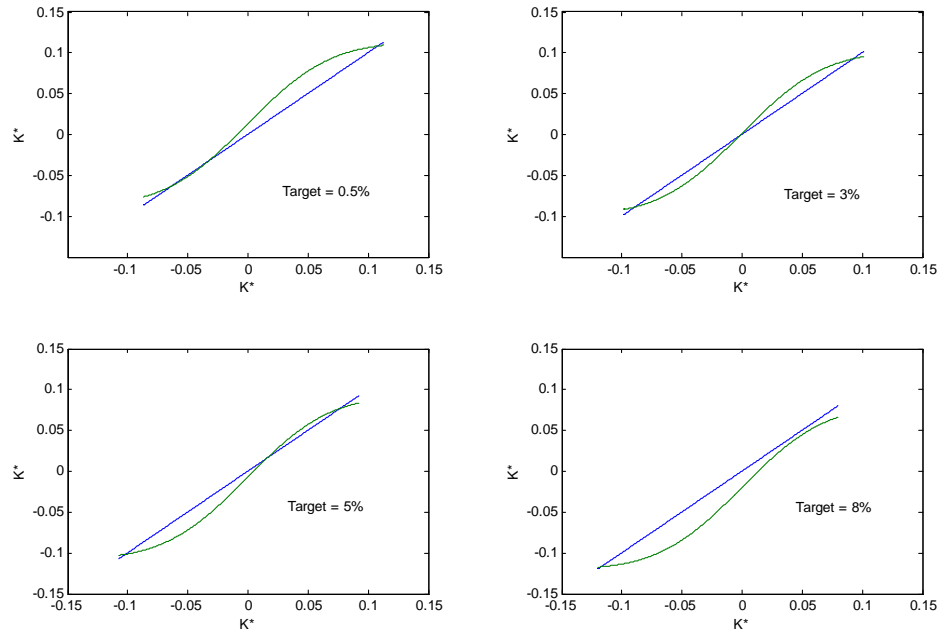


Figure 4: No-Common Knowledge  $K^*$ -Equilibrium

$$(s^p = 0, A = .5, \frac{\sigma}{\sigma_p} = \frac{.60}{.18}, c = .5, \rho = .2)$$

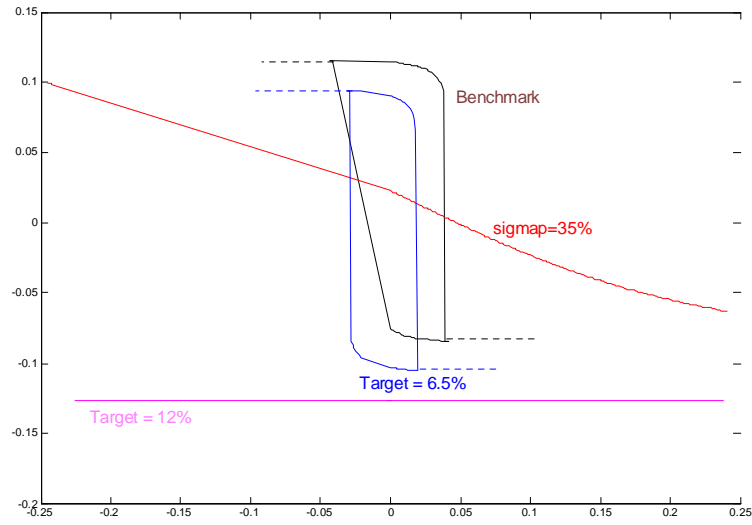


Figure 5: Extremes  $k^*$ -Equilibrium as function of  $s^p$

Vertical axis: Critical  $k^*$ . Horizontal axis: Public signal ( $s^p$ )

(Figure4's parameters are considered,  $\pi_a=.02$  for the benchmark case)

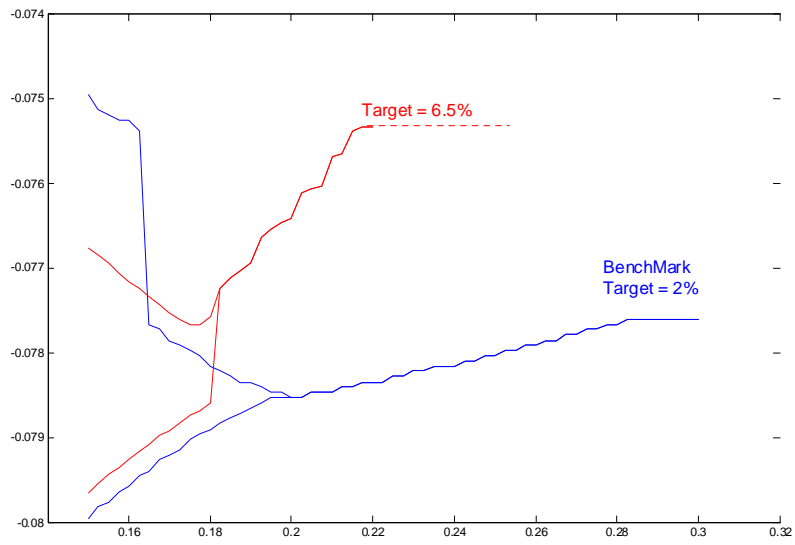


Figure 6: Welfare Cost in the Extreme Equilibria and Transparency

Vertical axis: Inflation welfare expected cost. Horizontal axis: Standard deviation of the public signal

(The Figure4's parameters are considered )

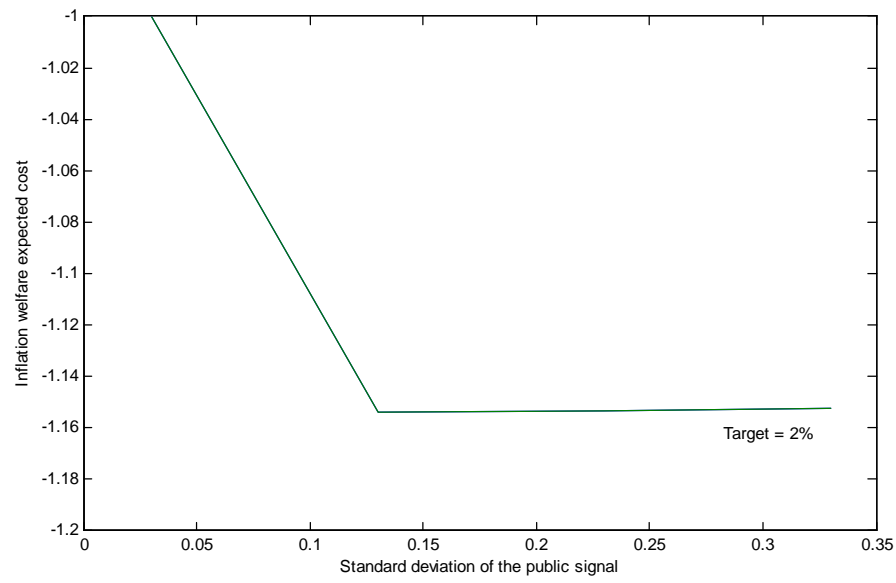


Figure 7: Welfare Cost in the Extreme Equilibria and Transparency for strong economy ( $k_o = 30$ )

(Again, The Figure4's parameters are considered )

## 6 Appendix

**Proof.** of proposition 1: The central bank from economy  $(A, B)$  solves the following problem:

$$\begin{aligned}\pi_a^* &= \arg \max_{\pi_a \in [0, A]} E [v(\pi_a, k)]; k \sim U[0, B] \\ v(\pi_a, k) &= \max_{\pi \geq 0} \left[ A(\pi - \pi_e(\pi_a)) - \frac{\pi^2}{2} - c(\pi_a, \pi, k) \right] \\ c(\pi_a, \pi, k) &= \begin{cases} 0 & \text{if } \pi_a = \pi \\ k & \text{if } \pi_a \neq \pi \end{cases}\end{aligned}$$

and it is easy to check that,

$$\begin{aligned}\pi_e &= \alpha \pi_a + (1 - \alpha) A \\ \alpha(\pi_a) &= \max \left\{ 1 - \frac{1}{B} \left[ A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]; 0 \right\} \\ E[k | \pi_a \neq \pi] &= \frac{A(A - \pi_a)}{2} + \frac{\pi_a^2}{4} - \frac{A^2}{4} \\ \alpha(\pi_a^*) &> 0\end{aligned}$$

It follows that:

$$\pi_a^* = \arg \max_{\pi_a} \frac{1}{B} \left[ B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right] \left( \frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) - \frac{3A^2}{4} - \frac{\pi_a^2}{4} + \frac{A\pi_a}{2}$$

The equilibrium,  $\pi_a^*$ , must solve:

$$\begin{aligned}u(\pi_a^*) &= v(\pi_a^*) \\ u(\pi_a) &\equiv \frac{B}{2} (A - \pi_a) \\ v(\pi_a) &\equiv \left( \frac{A + \pi_a}{2} \right) \left( B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right) - \left( \frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) (A - \pi_a)\end{aligned}$$

Since  $\left[ \frac{AB}{2} - A^3 = v(0) < u(0) = \frac{AB}{2} \right]$ ,  $[AB = v(A) > u(A) = 0]$  and  $[u'(\cdot) < 0]$ ; and

$v'(\cdot) > 0 \forall (\pi_a \leq A)$ ], then the equilibrium  $\pi_a^* \in (0, A)$  exists and is unique for any  $(A > 0, B > 0)$ .

Now, defining  $D(\pi_a^*) \equiv \pi_a^* - A(1 - \alpha(\pi_a^*))$ , it is easy to check that  $\text{sign}(D(\pi_a^*)) = \text{sign}(2B\pi_a^* + 2A^2\pi_a^* - A^3 - A(\pi_a^*)^2)$ .  $D(\cdot)$  is positive for  $\pi_a = A$  and negative for  $\pi_a = 0$ . Next we will show that  $D(\pi_a^*)$  is always positive, since  $\pi_a^*$  is bounded below by some positive value and  $D'(\cdot)$  is positive for  $\pi_a \in [0, A]$ . From  $\text{sign}(u(\pi_a) - v(\pi_a)) = \text{sign}((\pi_a)^3 + 2A^3 - \pi_a(3A^2 + 2B))$  we conclude that  $\pi_a^*$  solves  $(\pi_a^*)^3 + 2A^3 - \pi_a^*(3A^2 + 2B) = 0$ . Then, we can set  $\pi_a^* = \frac{2A^3 + (\pi_a^*)^3}{(3A^2 + 2B)}$ . Since  $D(\pi_a^*)$  is positive whenever  $\pi_a^* \geq \frac{A^3}{2B + A^2}$ , we must check if  $\left(\frac{2A^3 + (\pi_a^*)^3}{(3A^2 + 2B)} \geq \frac{A^3}{2B + A^2}\right)$  holds. This is straightforward for any  $(B \geq \frac{A^2}{2})$ . Now, if we set  $\pi_a = \frac{A^3}{B}$ , we have  $\text{sign}(u(\pi_a) - v(\pi_a)) = \text{sign}\left(\frac{A^4}{B^2} - 3\right)$ . We conclude that  $\frac{A^3}{B}$  is a lower bound to  $\pi_a^*$  whenever  $(B < \frac{A^2}{\sqrt{3}})$ . In this case,  $\pi_a^* \geq \frac{A^3}{B} \geq \frac{A^3}{2B + A^2}$  and  $D(\pi_a^*) > 0$  again. Since  $\left(\frac{A^2}{\sqrt{3}} > \frac{A^2}{2}\right)$ , we conclude that  $\pi_a^* \geq A(1 - \alpha(\pi_a^*))$  for any  $(A > 0, B > 0)$ . ■

**Proof.** of proposition 2: The target is fulfilled whenever  $w_a = k + h(\alpha) - x \geq 0$ , with  $x(\pi_a) \equiv \left[A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}\right]$ . The region for which the target  $(\pi_a)$  may induce multiple equilibria expectations is given by the interval  $[K^d, K^u]$ , where:

$$\begin{aligned} K^u(\pi_a, \underline{\alpha}) &= \inf \{k \in \mathbb{R} \mid (-x + k + h(\underline{\alpha})) \geq 0\} = x(\pi_a) - h(\underline{\alpha}) \\ K^d(\pi_a, \bar{\alpha}) &= \sup \{k \in \mathbb{R} \mid (-x + k + h(\bar{\alpha})) \leq 0\} = x(\pi_a) - h(\bar{\alpha}) \\ \bar{\alpha} &= \min \left\{ \frac{n - K^d}{\epsilon}; 1 \right\} \text{ if } [K^d, K^u] \cap [n - \epsilon, n] \neq \emptyset \\ \underline{\alpha} &= \max \left\{ \frac{n - K^u}{\epsilon}; 0 \right\} \text{ if } [K^d, K^u] \cap [n - \epsilon, n] \neq \emptyset \\ \bar{\alpha} &= \underline{\alpha} = 0 \text{ if } K^d > n \\ \bar{\alpha} &= \underline{\alpha} = 1 \text{ if } K^u < n - \epsilon \end{aligned}$$

There are five possible cases for the “[ $K^d, K^u$ ]-position” related to the support  $[n - \epsilon, n]$ , as follows:

Case	Exist $\Leftrightarrow$	$K^d$	$K^u$	Equilibrium
1	$x \in [n + h(0), n - \epsilon + h(1)]$	$\in [n - \epsilon, n]$ and $K^u = K^d$		$\alpha \in [0, 1]$
2	$x \in [n + h(0), n - \epsilon + h(1)]$	$\in [n - \epsilon, n]$	$> n$	$\alpha \in [0, 1]$
3	$x \in [n + h(0), n - \epsilon + h(1)]$	$< n - \epsilon$	$\in [n - \epsilon, n]$	$\alpha \in [0, 1]$
4	$n - \epsilon + h(1) > x$	$K^u < n - \epsilon$		$\alpha = 1$
5	$n + h(0) < x$	$K^d > n$		$\alpha = 0$

considering  $(h(1) - h(0) \geq \epsilon)$ . Otherwise, cases 2 and 3 do not exist and for cases 1, 4 and 5 we set  $x \in [n - \epsilon + h(1), n + h(0)]$  instead of setting  $x \in [n + h(0), n - \epsilon + h(1)]$ .

■

**Proof.** of proposition 3:

Since  $\left(\frac{dx}{d\pi_a}\right) = \pi_a - A$ , increasing the target is equivalent to reduce  $x$ . From  $\Psi(k^*, x) \equiv \Phi^{-1}\left(\frac{k^* + \rho - x}{\rho}\right) - \frac{\sigma}{\sigma_p^2} [k^*] = -\frac{\sigma}{\sigma_p^2} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\tau}}$  we conclude that  $\Psi(., x)$  is increasing in  $k^*$  for every  $s^p$  if  $\frac{\sigma \rho}{\sigma_p^2} \leq \sqrt{2\pi}$ . Reduction in  $x$  must be compensated by reduction in  $k^*$  in order to keep  $\left[-\frac{\sigma}{\sigma_p^2} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\tau}} = \Psi(k^*, x)\right]$  valid. The region over the  $\tilde{k}$ -support where the target is fulfilled increases for all  $(s^p)$  and the size of attack decreases as  $s^*(s^p)$  decreases.

$\Psi(., x)$  will be decreasing in  $k^*$  for *some* possible equilibrium  $\bar{k}^*(s^p)$  whenever  $\frac{\sigma \rho}{\sigma_p^2} > \sqrt{2\pi}$ . In this case, reduction in  $x$  must be compensated by an increasing in  $\bar{k}^*$  in order to keep  $\left[-\frac{\sigma}{\sigma_p^2} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\tau}} = \Psi(\bar{k}^*, x)\right]$  valid. So, an increase in the target may imply an increasing in  $\bar{k}^*$ ,  $s^*(\bar{k}^*)$ , and an increase in the size of attack. ■